Asymmetric Heat Flow in Mesoscopic Magnetic System

Keiji Saito

Department of Physics, Graduate School of Science, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan

The characteristics of heat flow in a coupled magnetic system are studied. The coupled system is composed of a gapped chain and a gapless chain. The system size is assumed to be quite small so that the mean free path is comparable to it. When the parameter set of the temperatures of reservoirs is exchanged, the characteristics of heat flow are studied with the Keldysh Green function technique. The asymmetry of current is found in the presence of a local equilibrium process at the contact between the magnetic systems. The present setup is realistic and such an effect will be observed in real experiments. We also discuss the simple phenomenological explanation to obtain the asymmetry.

KEYWORDS: quantum thermal conduction, heat flow control, quantum magnetic systems

1. Introduction

Recently, quantum heat transport in magnetic materials has attracted much interest.[1–7] In many magnetic materials, a magnetic contribution is dominant in heat transport in comparison with phononic heat transport. Generally the thermal conductivity is sensitive to magnetic phase transitions. Therefore, measurements of heat flow provide information on magnetic properties.[3] On the other hand, magnetic materials have also provided fundamental information on heat transport in nonequilibrium statistical mechanics. For example, which types of magnetic system show a diffusive transport in the thermodynamic limit? In an isotropic Heisenberg chain, the heat flow is a conserved quantity, and ballistic heat transport is theoretically expected.[8] Generally integrable systems show a divergent conductivity.[8,9] In Sr_2CuO_3 and $CuGeO_3$, which are well described by the isotropic Heisenberg chain, the ballistic feature of heat transport is actually confirmed by measuring an unusually large mean free path ($\sim$ 500-1000 times the lattice constant).[1–3] Interestingly, the Heisenberg chain with extra perturbation terms also shows very large conductivity, which is shown for many materials such as the spin-Ladder systems $(Sr,La,Ca)_{14}Cu_{24}O_{41}$ and $La_5Ca_9Cu_{24}O_{41}$.[5–7] Although in real materials, macroscopic heat transport in the thermodynamic limit cannot be ballistic because of the existence of spin-phonon interaction, impurities and so on, it is still of academic interest to study the thermodynamic behavior of magnetic heat transport. Therefore, both theoretically and experimentally, the properties of energy transport in the thermodynamic limit have been intensively studied.[10–18]

In this paper, however, we do not study the thermodynamic limit. We focus on mesoscopic-scale systems, where mean free path is comparable to system size. Therefore, phononic heat transport is negligible at very low temperatures, and magnetic heat transport is regarded as ballistic. We study a coupled magnetic systems and consider how the heat flow can be controlled. In this case, two magnetic materials are connected to each other, and it is assumed that one magnetic system is gapless, and the other is gapped. In this case, there exists no magnetic interaction between the two magnetic chains. Therefore, the heat is conveyed through phonons at the surface of contact between them. Here, we ask what should be expected for heat flow in this setup.

A similar setup has been studied in classical anharmonic chains, where left and right chains have different phononic mode distributions. At the left and right edges, thermal reservoirs at different temperatures are attached. Terraneo et al. numerically showed that asymmetric heat flow is observed when the temperatures at both edges are exchanged.[20,21] This phenomenon is observed only for the regime far from equilibrium, and cannot be explained within the linear response theorem. Recently, it has been desired to demonstrate this phenomenon in real experiments.[?] In this paper, we propose a promising experimental setup to observe asymmetric heat flow using the above coupled magnetic system. It is emphasized that our setup is based on a *mesoscopic quantum magnetic system* so that we have a wide choice of magnetic material.

Here, we theoretically analyze heat flow using the bosonization technique.[24] The bosonized Hamiltonian is convenient because it guarantees ballistic energy flow, which is expected in a mesoscopic regime.[19] We adopt the Keldysh Green function technique for the boson operators to study the nonequilibrium stationary state. We show that asymmetric heat flow is actually observed only when the thermalization of phonons occurs, which is normally expected in real experiments.

This paper is organized as follows. In §2, we introduce the model, and analyze the stationary state using the Keldysh Green function method in §3. In §4, another analysis to interpret the phenomenon is introduced. The summary and a brief discussion are given in §5.

2. Coupled Magnetic Systems

We consider the composite mesoscopic spin-$\frac{1}{2}$ system, which is composed of a gapless chain and a gapped chain. This attachment of two different mesoscopic magnetic systems means that heat transport between magnetic chains is possible by spin-phonon interaction at the surface of contact between them. Here, it is assumed that the mean free paths in both magnetic systems are comparable to system size. These setups are depicted in Fig. 1(a). We now use the following type of Hamiltonian to investigate the heat flow at the nonequilibrium stationary state:

$$\mathcal{H} = \mathcal{H}_{\mathrm{L}} + \mathcal{H}_{\mathrm{R}} + \mathcal{H}_p + \mathcal{H}' \tag{1}$$

$$\mathcal{H}_{\mathrm{p}} = \sum_k \omega_k b_k^\dagger b_k \tag{2}$$

$$\mathcal{H}' = \sum_k \lambda_{\mathrm{L}k} s^z_{\mathrm{L},1} \left(b_k + b_k^\dagger\right) + \sum_k \lambda_{\mathrm{R}k} s^z_{\mathrm{R},1} \left(b_k + b_k^\dagger\right) \tag{3}$$

Here $\mathcal{H}_\alpha$ ($\alpha = \mathrm{L}$ and R) is the Hamiltonian of the left and right magnetic systems, and $\mathcal{H}_{\mathrm{p}}$ expresses the phonon part. The operators $s^z_{\mathrm{L},1}$ and $s^z_{\mathrm{R},1}$ are the z-component of spins at the edges of the left and right magnetic systems, and they are assumed to interact with phonons localized at the contact between magnetic systems. These decompositions are schematically shown in Fig. 1(b). Here, the isotropic Heisenberg chain is assumed to be the gapless chain, but no concrete gapped chain is assumed at present.

We treat mesoscopic-scale systems so that mean free path is comparable to material size in both magnetic materials, where heat is conveyed without scattering within the material. In this case, the effective Hamiltonian is well described by the bosonization form of the spin systems. Generally, bosonized Hamiltonians satisfy a ballistic feature of transport because the bosonized heat flow operator is commutable with the Hamiltonian.[19] Thus, the bosonized form of the spin system automatically satisfies the characteristics of the mesoscopic nature of the magnetic systems. We adopt the following bosonized Hamiltonian:

$$\mathcal{H}_{\mathrm{L}} = \frac{u}{2}\int_{x_0}^{\infty} dx\, \left[\Pi_{\mathrm{L}}^2(x) + \left(\partial_x \Phi_{\mathrm{L}}(x)\right)^2\right] \tag{4}$$

$$\mathcal{H}_{\mathrm{R}} = \frac{u}{2}\int_{x_0}^{\infty} dx\, \left[\Pi_{\mathrm{R}}^2(x) + \left(\partial_x \Phi_{\mathrm{R}}(x)\right)^2 + g_0 \Phi_{\mathrm{R}}^2(x)\right] \tag{5}$$

$$\mathcal{H}' = \sum_{\alpha=\mathrm{L,R}} \left(\frac{1}{\sqrt{\pi}}\partial_x \Phi_\alpha(x_0) + \sum_k \frac{\lambda_{\alpha k}}{2}\left(b_k + b_k^\dagger\right)\right)^2 \tag{6}$$

The boson operators $\Phi_\alpha(x)$ and $\Pi_{\alpha'}(x')$ satisfy the commutation relation $[\Phi_\alpha(x), \Pi_{\alpha'}(x')] = \mathrm{i}\delta_{\alpha,\alpha'}\delta(x-x')$. The operator $\frac{1}{\sqrt{\pi}}\partial_x\Phi_\alpha(x_0)$ is the z-component of spin at the position x_0 in the αth spin chain. In the interaction Hamiltonian $\mathcal{H}'$, the quadratic terms of bosons are added only to avoid an unbounded state. For simplicity of calculation, we adopt the bilinear form for the gapped Hamiltonian, which can be regarded as the approximated form of the sine-Gordon Hamiltonian $\mathcal{H}_{\mathrm{R}}$.[24,25] The sine-Gordon Hamiltonian can be mapped from many gapped magnetic chains. The present gapped Hamiltonian gives the dispersion relation with the energy gap $\Delta = u\sqrt{g_0}$. We believe that this simplification does not cause different qualitative results because correct excitation spectrum and excitation eigenfunctions are not necessary for the phenomenon described in this paper.

3. Heat Flow

In this section, we calculate the heat flow at the steady state. As explained in the previous section, the mesoscopic nature of current is represented by the ballistic heat flow in the bosonized Hamiltonian. The next task is the realization of the nonequilibrium steady state of

the system. For simplicity of calculation, we assume an infinite size for the left and right magnetic systems, instead of connecting thermal reservoirs at the edges of the magnetic systems. By taking the infinite size, we can realize a nonequilibrium steady state as follows (see Fig. 1(b)). We assume that both magnetic systems are initially separated and are in equilibrium with the temperatures T_{L} and T_{R}. Next, these systems are adiabatically connected with the phonon system, and a steady state is realized in the long time limit. The steady state can be analyzed by the Keldysh Green function technique.[23] To this end, the following operators are defined

$$X_\alpha = \sum_k \lambda_{\alpha k}\left(b_{\alpha k}+b^\dagger_{\alpha k}\right), \quad Y_\alpha = \mathrm{i}\sum_k \lambda_{\alpha k}\left(b_{\alpha k}-b^\dagger_{\alpha k}\right) \quad \alpha = \mathrm{R} \text{ or } \mathrm{L} \tag{7}$$

We further assume the simple form of coupling for spin-phonon interaction as $\lambda_{\mathrm{L}k} = \lambda_{\mathrm{R}k} = \lambda_k$. We do not discriminate between the indices α of operators X_α and Y_α, and simply write X and Y, respectively. The heat flow is expressed by the Keldysh lesser Green function

$$\begin{aligned}\langle J\rangle &= \langle \frac{\mathrm{i}}{\sqrt{\pi}}\partial_x\Phi_1(x_0,t)Y(t)\rangle = \frac{\mathrm{i}}{\sqrt{\pi}}G^<_{Y,\partial\Phi_1}(t,t) \\ &= -\frac{1}{2\pi^2}\int_{-\infty}^{\infty} d\Omega\ \mathrm{Im}\left\{G^r_{Y,X}(\Omega)g^<_{\partial\Phi_{\mathrm{L}},\partial\Phi_{\mathrm{L}}}(\Omega)+G^<_{Y,X}(\Omega)g^a_{\partial\Phi_{\mathrm{L}},\partial\Phi_{\mathrm{L}}}(\Omega)\right\}\end{aligned} \tag{8}$$

Throughout this paper, we set $\hbar$ to be unity. Here $G^r_{Y,X}(\Omega)$ and $G^<_{Y,X}(\Omega)$ are the Fourier transforms of the retarded and lesser Green functions:

$$\begin{aligned}G^r_{Y,X}(t,t') &= -\mathrm{i}\Theta(t-t')\langle\left[Y(t),X(t')\right]\rangle \\ G^<_{Y,X}(t,t') &= -\mathrm{i}\langle X(t')Y(t)\rangle\end{aligned}$$

The functions $g^<_{A,B}(\Omega)$ and $g^{r,a}_{A,B}(\Omega)$ are the Fourier transforms of the nonperturbed Green function:

$$\begin{aligned}g^<_{A,B}(t,t') &= -\mathrm{i}\langle B(t')A(t)\rangle_0 \\ g^{r,a}_{A,B}(t,t') &= \mp\mathrm{i}\Theta\left(\pm(t-t')\right)\langle[A(t),B(t')]\rangle_0\end{aligned}$$

Here $\langle ...\rangle_0$ denotes an average over a nonperturbed Hamiltonian. The nonperturbed Green functions are readily calculated as

$$\begin{pmatrix} g^{r,a}_{X_kX_k}(\Omega), & g^{r,a}_{X_kY_k}(\Omega) \\ g^{r,a}_{Y_kX_k}(\Omega), & g^{r,a}_{Y_kY_k}(\Omega)\end{pmatrix} = \frac{\lambda_k^2/\pi}{(\Omega\pm\mathrm{i}\varepsilon)^2-\omega_k}\begin{pmatrix}\omega_k & -\mathrm{i}\left(\Omega\pm i\varepsilon\right) \\ \mathrm{i}\left(\Omega\pm\mathrm{i}\varepsilon\right) & \omega_k\end{pmatrix}, \tag{9}$$

$$\mathrm{i}g^<_{\partial\Phi_\alpha,\partial\Phi_\alpha}(\Omega) = \begin{cases} 2\frac{\sqrt{\Omega^2-\Delta_\alpha^2}}{v^2}\frac{\mathrm{sgn}(\Omega)}{\mathrm{e}^{\beta\Omega}-1} & \text{for} \quad |\Omega|>\Delta_\alpha \\ 0 & \text{for} \quad |\Omega|\le\Delta_\alpha\end{cases}, \tag{10}$$

$$g^{r,a}_{\partial\Phi_\alpha,\partial\Phi_\alpha}(\Omega) = \begin{cases} \mp\frac{\mathrm{i}}{v^2}\mathrm{sgn}(\Omega)\sqrt{\Omega^2-\Delta_\alpha^2} & \text{for} \quad |\Omega|>\Delta_\alpha \\ \frac{\sqrt{\Delta_\alpha^2-\Omega^2}}{v^2} & \text{for} \quad |\Omega|\le\Delta_\alpha\end{cases}, \tag{11}$$

where α is L or R, and $\Delta_{\mathrm{L}} = 0$, and $\Delta_{\mathrm{R}} = u\sqrt{g_0}$. Here, we neglected the effects of the quadratic terms of the bosons in the Hamiltonian (6), which is not expected to have a large contribution to the stationary state.

In realistic systems, it is natural to expect that the phonon system at the contact between magnetic systems becomes a local equilibrium state at the steady state. To realize the thermalization of the phonon part, we adopt the so-called self-consistent reservoir (SCR) by connecting an extra infinite system, which can be regarded as the reservoir with some temperature T_{S} at the initial time. The temperature T_{S} is determined under the condition that heat flow vanishes between the reservoir and the phonon system at the stationary state as depicted in Fig. 2(a). Such an extra system induces the thermalization of phonons, and the degree of a local equilibrium state is controlled by the coupling strength between the SCR and the phonon system.[26] Since this extra system is simply adopted only to induce the thermalization of the phonon system, we have many choices for the system. For simplicity, we now use the spin system for the SCR, which is the same form of $\mathcal{H}_{\mathrm{L}}$ because all Green functions are already known. The interaction Hamiltonian where the phonon couples with the SCR is

$$\mathcal{H}_{\mathrm{SCR-P}} = \lambda \left(\frac{1}{\sqrt{\pi}} \partial_x \Phi_{\mathrm{SCR}}(x_0) + \sum_k \frac{\lambda_{\alpha k}}{2} \left(b_k + b_k^\dagger \right) \right)^2, \tag{12}$$

where $\frac{1}{\sqrt{\pi}}\partial_x\Phi_{\mathrm{SCR}}(x_0)$ is the z-component of spin at x_0 in the SCR, and λ is the coupling strength between SCR and the phonon system. The strength of λ controls the degree of local equilibrium. The spin system for the SCR is the same form as that $\mathcal{H}_{\mathrm{L}}$; $\mathcal{H}_{\mathrm{SCR}} = \frac{u}{2}\int_0^\infty dx \left[\Pi_{\mathrm{SCR}}^2(x) + (\partial_x \Phi_{\mathrm{SCR}}(x))^2 \right]$. The Green functions $G^r_{Y,X}(\Omega)$ and $G^<_{Y,X}(\Omega)$ in the formula (8) are calculated by the Keldysh relation[23] with the nonperturbed Green functions of the phonon system, two magnetic systems and the system of SCR.

Let us now see the properties of heat flow in these setups. Two cases are investigated, i.e., $(T_{\mathrm{L}}, T_{\mathrm{R}}) = (\Delta T, 0)$ and $(T_{\mathrm{L}}, T_{\mathrm{R}}) = (0, \Delta T)$. The heat flows in the former and the latter cases are written as J_{L} and J_{R}, respectively. In these two cases, the behavior of heat flow is investigated for various coupling strengths λ. Numerical calculation is self-consistently performed to satisfy the condition of SCR. Here, we took 1000 boson numbers for the phonon system with the ohmic spectral density. The energy gap is $\Delta = 3.0$ in the unit of u. The temperature dependences of heat flow are shown in Fig. 2(b) for various coupling strength λ. As shown in Fig. 2(b), in the case of $\lambda = 0$, no asymmetry of heat flow is observed. However as λ increases, asymmetry appears. For a larger λ, the amplitude of heat flow decreases, which is due to the scattering effect of energy by the SCR. In the case of finite λ, $|J_{\mathrm{R}}|$ is always larger than $|J_{\mathrm{L}}|$.

4. Extreme Model to understand Mechanisms

As shown in the previous section, asymmetric current can occur in the presence of the thermalization of phonons. This means that the phonon system has some temperature de-

fined in the local equilibrium state. In this section, we consider a simple phenomenological explanation of the asymmetric heat flow regarding the phonon system as *the phonon reservoir*.

To this end, the following two problems are considered. i) First, what amount of the stationary heat flow is obtained in one boson system when it is coupled to two different reservoirs with different temperatures ? Here the boson system is the main system of interest. ii) Second, the following situation is studied using the general formula of heat flow derived in the first problem. Suppose that the different two boson systems sandwich a phonon reservoir with some temperature, $T_{\rm S}$. At the edges of the boson system, other phonon reservoirs are connected with the temperatures $T_{\rm L}$ and $T_{\rm R}$ (see the schematic explanation of the situation in the inset in the Fig. 3). What is the characteristic of heat flow, when the temperature T_S is determined under the condition that both stationary heat flows within the boson systems are equal ? This extreme situation actually explains the asymmetry of heat flow.

We study the first problem by considering the dynamics for the Hamiltonian;

$$\mathcal{H}_{\rm tot} = \mathcal{H} + \sum_{\alpha=1,2} \nu \left(X_\alpha - \sum_k \gamma_{\alpha,k}(a_{\alpha,k} + a^\dagger_{\alpha,k}) \right)^2 + \sum_{\alpha,k} \omega_k a^\dagger_{\alpha,k} a_{\alpha,k}, \tag{13}$$

$$\mathcal{H} = \sum_k \Omega_k b^\dagger_k b_k, \tag{14}$$

$$X_\alpha = \sum_k \mu_{\alpha,k} \left(b^\dagger_k + b_k \right), \tag{15}$$

where $\mathcal{H}$ is the boson system of interest and the operators $a_{\alpha,k}$ and $a^\dagger_{\alpha,k}$ are the annihilation and creation phonon operators in the phonon reservoirs. The operators X_1 and X_2 are the system's operators attached to the phonon reservoirs. We assume that the phonon reservoirs have spectral densities $I_\alpha(\omega)$ and temperatures T_α.

By standard projection operator technique,[27] we obtain the Redfield-type master equation for a reduced density operator[28,29] up to the second order of ν

$$\frac{\partial \rho(t)}{\partial t} = -{\rm i}\,[\mathcal{H}, \rho(t)] - \nu^2 \mathcal{L}_1 \rho(t) - \nu^2 \mathcal{L}_2 \rho(t) \tag{16}$$

$$\mathcal{L}_\alpha \rho(t) = [X_\alpha, R_\alpha \rho(t)] + [X_\alpha, R_\alpha \rho(t)]^\dagger \tag{17}$$

$$R_\alpha = \sum_k \mu_k I_\alpha(\Omega_k) n_\alpha(\Omega_k) \left({\rm e}^{\beta_\alpha \Omega_k} b_k + b^\dagger_k \right) \tag{18}$$

Here, $n_\alpha(\Omega_k)$ is the Bose distribution, $n_\alpha(\Omega_k) = 1/({\rm e}^{\beta_\alpha \Omega_k} - 1)$. When we expand the stationary density matrix as $\rho_{\rm st} = \rho^{(0)}_{\rm st} + \nu^2 \rho^{(1)}_{\rm st} + \cdots$, we obtain the zeroth order of density matrix as

$$\rho^{(0)}_{\rm st} = \Pi_k \frac{{\rm e}^{-B_k \Omega_k b^\dagger_k b_k}}{Z}, \qquad B_k = \frac{1}{\Omega_k} \ln \left[\frac{\sum_{\alpha=1,2} \mu^2_{\alpha,k} I_\alpha(\Omega_k) n_\alpha(\Omega_k) {\rm e}^{\beta_\alpha \Omega_k}}{\sum_{\alpha=1,2} \mu^2_{\alpha,k} I_\alpha(\Omega_k) n_\alpha(\Omega_k)} \right]. \tag{19}$$

The heat flow operator from the first reservoir to system is derived from the continuity equation of energy

$$\hat{j} = \nu^2 \mathcal{H} \mathcal{L}_1. \tag{20}$$

Thus, using the zeroth order stationary density matrix $\rho_{\mathrm{st}}^{(0)}$, we obtain the leading order of average current $\langle j \rangle = \nu^2 \mathrm{Tr}\left(\mathcal{H}\mathcal{L}_1\rho_{\mathrm{st}}^{(0)}\right)$. The average current $\langle j \rangle$ is readily calculated,

$$\langle j \rangle \quad = \quad \nu^2 \int d\varepsilon \mathcal{D}(\varepsilon)\varepsilon \left[\frac{\mu_1(\varepsilon)I_1(\varepsilon)\mu_2(\varepsilon)I_2(\varepsilon)}{\mu_1(\varepsilon)I_1(\varepsilon)+\mu_2(\varepsilon)I_2(\varepsilon)}\right](n_2(\varepsilon)-n_1(\varepsilon)), \tag{21}$$

where $\mathcal{D}(\varepsilon)$ is the density of states at the energy ε, and $\mu_\alpha(\varepsilon) = \sum_k \mu_{\alpha,k}^2 \delta(\varepsilon - \Omega_k)$. The function of $\mu_{\alpha,k}$ depends on detailed systems. Below we consider the special case with $\mu_{1,k}^2 = \mu_{2,k}^2$ for simplicity. This case is actually realized in some boson systems.[30] In this case, eq.(21) becomes simpler because of $\mu_1(\varepsilon) = \mu_2(\varepsilon)$.

We are now in the position to discuss the second problem and clarify the essential mechanism of asymmetric heat flow. Suppose that the different boson systems are connected via a phonon reservoir with temperature T_{S}, and are also attached to phonon reservoirs with different temperatures, T_{L} and T_{R} (see the inset in Fig. 3). We call the heat flow within the left and right boson systems as $j(T_{\mathrm{L}}, T_{\mathrm{S}})$ and $j(T_{\mathrm{S}}, T_{\mathrm{R}})$, respectively. At the stationary state, it is demanded that both currents are equal, i.e.,

$$j(T_{\mathrm{L}}, T_{\mathrm{S}}) = j(T_{\mathrm{S}}, T_{\mathrm{R}}), \tag{22}$$

from which the temperature T_{S} is self-consistently determined. Note that the heat flow is measured under these conditions.

We now demonstrate that the asymmetry of current actually occurs when the energy densities of the left and right systems, $\mathcal{D}_{\mathrm{L}}(\varepsilon)$ and $\mathcal{D}_{\mathrm{R}}(\varepsilon)$ are different. We consider the simplest case of $\mathcal{D}_{\mathrm{L}}(\varepsilon) = d_0\varepsilon$ and $\mathcal{D}_{\mathrm{R}}(\varepsilon) = d_0\sqrt{\varepsilon^2+\Delta^2}$. In this case, we calculate the energy flow J_{L} for the temperature set $(T_{\mathrm{L}}, T_{\mathrm{R}}) = (\Delta T, 0)$, and J_{R} for $(T_{\mathrm{L}}, T_{\mathrm{R}}) = (0, \Delta T)$. In Fig. 3, we present the results of J_{R} and J_{L} under these conditions. Here, energy gap $\Delta = 3.0$, and we used the same phonon reservoir with ohmic spectral density. As shown, we find that the asymmetry appears in the heat flow, and the qualitative results in the previous section are reproduced.

5. Summary

We discussed the possibility of observation of asymmetric heat flow in a coupled magnetic material. We particularly focused on mesoscopic-scale magnetic systems, where mean free path is comparable to system size. We found that an asymmetric current is observed as a result of the thermalization of phonons at the contact between magnetic materials. The present experimental setup is realistic, and we have many choices of realistic magnetic materials. The asymmetry of heat flow is a key ingredient for heat pumping. It will be important to discuss a realistic setup of heat pumping and the efficiency,[31] although we do not discuss it here.

In a gapped spin chain, a phononic contribution may appear in realistic experiments. Actually in $CuGeO_3$, which shows spin-Peierls transition, there exists a crossover from magnetic heat transport to phononic heat transport when the temperature decreases across the spin-Peierls temperature.[3] The quantitative properties depend on spin-phonon interaction in the

bulk materials. Thus, to realize spin heat transport in all temperature regimes, we must find a suitable material with very weak spin-phonon interaction.

In the present study, we consider small magnetic systems, where the analysis becomes easy. In the case that mean free path is much smaller than material size, treatment is much more difficult than the present analysis because most gapped magnetic systems are nonintegrable systems, and use of the bosonized Hamiltonian may not be justified.[18] It will also be important to study such a region, because it is not clear whether observation of asymmetric heat flow requires the ballistic properties of current. To answer this question, real experiments would be very useful.

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29) K. Saito, S. Takesue and S. Miyashita: Phys. Rev. E **61** (2000) 2397.
30) For example, in case of the quantum harmonic chain whose Hamiltonian: $\mathcal{H} = \sum_{n=1}^{N} \frac{p_n^2}{2m} + \sum_{n=0}^{N} \frac{m\omega_0^2}{2}(x_{n+1} - x_n)^2$ $(x_0 = x_{N+1} = 0)$, X_1 and X_2 are x_1 and x_N. The functions $\mu_{1,k}$ and $\mu_{2,k}$ are $\mu_{1,k} = \sqrt{\frac{\hbar}{2(N+1)m\omega_0}} \frac{\sin k}{\sqrt{\sin(k/2)}}$, and $\mu_{2,k} = \sqrt{\frac{\hbar}{2(N+1)m\omega_0}} \frac{\sin(Nk)}{\sqrt{\sin(k/2)}}$, where $k = \pi\ell/(N+1)$ ($\ell =$

$1, 2, \cdots, N$). These functions satisfy $\mu_{1,k}^2 = \mu_{2,k}^2$.

31) D. Segal and A. Nitzan, cond-mat/0510262/.

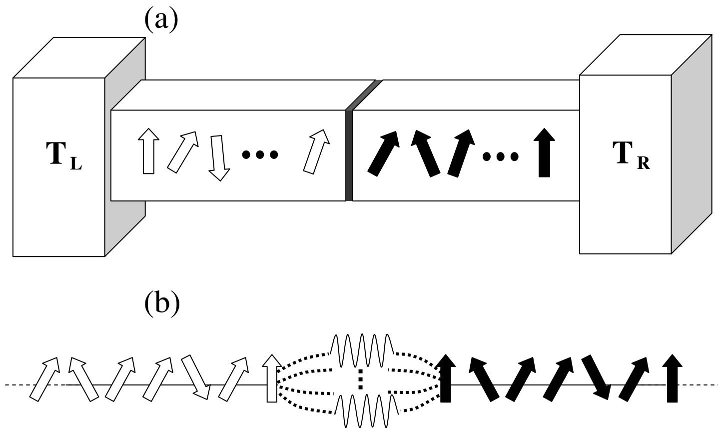

Fig. 1. (a): Experimental setup of coupled magnetic system. It is assumed that no magnetic interactions exist between the two magnetic systems. The heat can be conveyed by phonons at the surface of the contact. (b): Theoretical setup. At the contact between the magnetic systems, spin-phonon interaction exists.

Fig.1, Keiji Saito

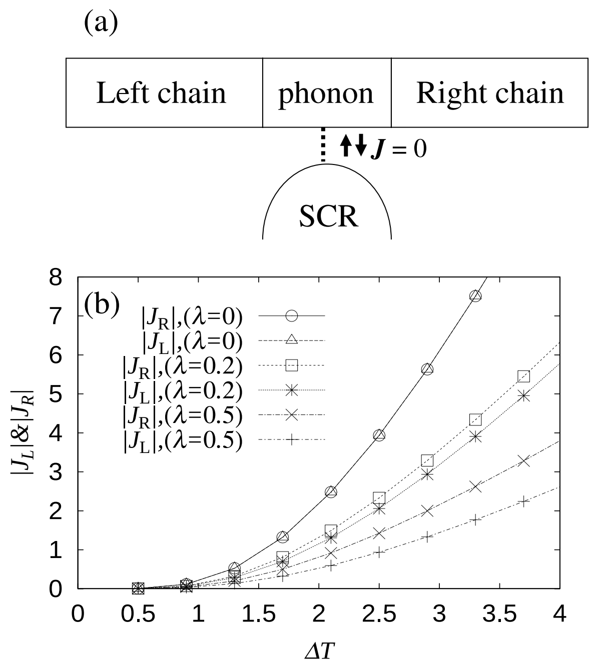

Fig. 2. (a) Schematic explanation of structure of self-consistent reservoir. Between the SCR and the phonon, there is no heat flow at the stationary state. Thermalization occurs at the phonon part with a finite coupling strength, λ. (b) $|J_{\rm L}|$ and $|J_{\rm R}|$ as functions of ΔT for various coupling strengths λ of SCR. Here, the energy gap of the right spin chain is 3.0 in the unit of u. The unit of the heat flow is $u^2/\hbar$.

Fig.2, Keiji Saito

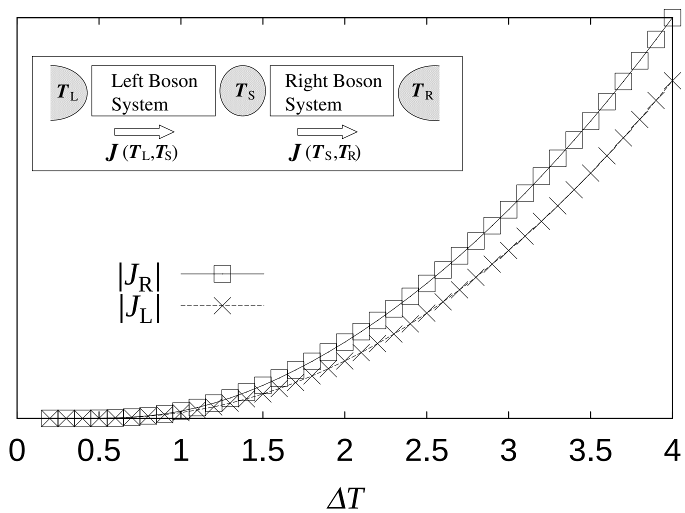

Fig. 3. Simple explanation of asymmetric heat flow using phonon reservoirs with temperatures $T_{\rm L}$, $T_{\rm S}$, and $T_{\rm R}$. The temperature $T_{\rm S}$ is determined by the condition $J(T_{\rm L}, T_{\rm S}) = J(T_{\rm S}, T_{\rm R})$

Fig.3, Keiji Saito